Control, Replenishment, and Stability of Life Support Systems

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A mathematical analysis of an ecological system consisting of a space cabin, crew, and life support system is outlined. Methods for predicting system life and for the evaluation of the merits of different resupply cycles are discussed. Primary consideration is given to the O_2-CO_2 cycle and the water cycle. The system is described by a set of differential equations with restraints very similar to those given in flight control problems. Control and design parameters are represented so that system stability and optimization with respect to such factors as weight, energy consumption, and reaction time to a temporary imbalance can be studied.

Introduction

THE question of whether a life support system is to be completely self-sufficient as launched, or whether it is to be replenished and, if so, on what cycle, must be answered on the basis of specific features of the space mission. Figure 1 illustrates a hypothetical cost comparison of such features. Launch costs are smallest when all supplies are put into orbit in the initial package by using some large standardized booster to its full potential. (The cost of the propulsion and electronics systems needed for the rendezvous operation are included.) However, this factor is partially compensated by decreasing on-site storage cost for resupply systems. When no resupply is available, the mission plan cannot be changed during the course of the operation. This lack of flexibility could be quite serious if the on-board equipment is found to be unsuitable to the conditions encountered, for example, some of the radiation counters in early satellites.

Similarly, a valuation must be made of the ability to replace inoperative life support components, provide medicine, and remove sick crew members. On the other hand, the complexity of the resupply operation and the possibility of a disastrous accident will counter the added flexibility. Finally, more working equipment, radiation shielding, or other alternate payload could be put into orbit if less life support weight were orbited in the initial launch package.

Clearly, such an analysis can be made only when a mission is specified in detail. The intent of this paper is not to carry out such an analysis. However, in order to schedule the resupply, if any, of a life support system, it is necessary to determine the changes that take place in the quantities of certain substances in the ecological system composed of the life support system, the crew, the cabin atmosphere, and any plants, animals, or material-consuming equipment that may be present. The determination of these changes is not all straightforward when regeneration of some of the substances occurs. The purpose of this paper is to outline the mathematical framework from which a general system analysis can be made so that, in addition to the determination of resupply requirements, there will result 1) a calculation of the output required of the various system components (in other words, a preliminary design); 2) a formulation of the control problem from which optimal control functions can be determined; 3) a means of determining the time required to bring the system back to a suitable balance in the case that a mishap occurs, and also the time required initially to put the system into operation; and 4) a means of making trade-off studies.

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The model is first formulated using a simple open-end system to illustrate the basic concepts, and then it is applied to a system that uses electrolysis of lithium carbonate to regenerate oxygen. The results of this analysis are then used in connection with stability and weight-power considerations for the lithium carbonate system.

Formulation of the Model

At any time t, the *state* of the ecological system is defined to be a set of mxn quantities $x_{ij}(t)$; $i = 1, \ldots, n$; $j = 1, \ldots, m$, where $x_{ij}(t)$ is the quantity of the ith material in the jth compartment at a given time. The materials considered include N_2, O_2, CO_2, H_2O, and certain contaminants such as CO, SO_2, CH_4, etc. The compartments considered are the atmosphere, the crew, CO_2 removal units, CO_2-O_2 exchangers, plants, animals, electrolysis tanks, storage tanks, etc. Certain of these compartments, notably the biological elements and the life support components, are considered as reactors or exchangers. For example, the crew, upon assimilating O_2, will produce CO_2 to be returned to the atmosphere. It will be convenient to measure material quantities in moles.

The schematic of a system that recovers water only is shown in Fig. 2 to illustrate these concepts. Components of the system which contain significant amounts of the material ingredients are shown as rectangles, and those components treated as reactors only have an ovoid shape. Figure 3 illustrates the system used to record the state variables, e.g., x_{12} in the second row and first column is the moles of oxygen in storage (varies with time).

The rates of material transference between the atmosphere and the other components, and between storage and other system components, are recorded in Figs. 4 and 5, respectively. Thus the $-Lx_{11}$ appearing in the first row and first column of Fig. 4 represents the leakage of oxygen from the cabin atmosphere to space, based on the assumption that leakage is proportional to the partial pressure of that constituent in the atmosphere. Similarly, the Y_1 and the Y_4 of row 2 represent the rates at which oxygen and nitrogen are released to the atmosphere from storage. The crew's production and consumption of various atmospheric constituents is shown in row 4.

Water is removed by blowing air over cooling coils at a rate such that Bx_{31} moles of water vapor pass through the coils per unit time. The amount of water passing out of the coils with the exhaust air is taken to be Bb (b is a function of the temperature of the coils and their shape), so that the number of moles of water vapor removed per minute is $B(x_{31} - b)$ as indicated in row 5, column 3.

Air is pumped through the CO_2 removal unit at rate such that Ax_{21} moles of CO_2 pass through the component per unit time. It is assumed that 100% of the CO_2 is removed from this air and hence the entry Ax_{21} in row 6 column 2. The entries of row 7 are obtained similarly.

The entries of Figs. 3, 4, and 5 are variable with time.

Each column of Figs. 4 and 5 determines a differential equation. Thus the first column of Fig. 4 determines the first equation below, and the first column of Fig. 5 determines the second equation ($\dot{R}_1 = 0$ temporarily):

$$\left.\begin{aligned}
dx_{11}/dt &= Y_1 - K_2 - Lx_{11} \\
dx_{12}/dt &= -Y_1 \\
dx_{21}/dt &= -(A + L)x_{21} + 0.85K_2 \\
dx_{22}/dt &= Ax_{21} \\
dx_{31}/dt &= K_3 - B(x_{31} - b) - Lx_{31} \\
dx_{32}/dt &= B(x_{31} - b) - K_4 + K_5 \\
dx_{41}/dt &= Y_4 - Lx_{41} \\
dx_{42}/dt &= -Y_4 \\
dx_{51}/dt &= -Cx_{51} + K_7 - Lx_{51}
\end{aligned}\right\} \qquad (1)$$

This set of simultaneous differential equations forms the first of three major features of the mathematical model of the ecological system. These equations, which describe the operation of the basic system, form a family of linear differential equations, and it is not difficult to obtain solutions in the form below (where $Y_1, K_2, \ldots$, are functions of time):

$$\left.\begin{aligned}
x_{11} &= \exp(-Lt)\left(\int_0^t \exp(Lt)(Y_1 - K_2)dt + x_{11}(0)\right) \\
x_{12} &= -\int_0^t Y_1 dt + x_{12}(0) \\
x_{21} &= \exp\left[-\int_0^t (A + L)dt\right]\left\{\int_0^t \exp\left[\int_0^t (A + L)dt\right] 0.85K_2 dt + x_{21}(0)\right\} \\
x_{22} &= \int_0^t \left(\exp\left[-\int_0^t (A + L)dt\right] \times A\left\{\int_0^t \exp\left[\int_0^t (A + L)dt\right]0.85K_2 dt\right\} \times dt + x_{21}(0)\right) dt + x_{22}(0) \\
x_{31} &= \exp\left[-\int_0^t (L + B)dt\right] \times \left\{\int_0^t \left(\exp\left[\int_0^t (L + B)dt\right]\right)(K_3 + Bb)dt + x_{31}(0)\right\} \\
x_{32} &= \int_0^t \left[Bx_{31}(t) - K_4 + K_5 + Bb\right]dt + x_{32}(0) \\
x_{41} &= \left[\exp(-Lt) - 1\right]\left\{\int_0^t \left[\exp(Lt) - 1\right] \times Y_4 dt + x_{41}(0)\right\} \\
x_{42} &= -\int_0^t Y_4 dt + x_{42}(0) \\
x_{51} &= \exp\left[-\int_0^t (C + L)dt\right] \times \left\{\int_0^t \exp\left[\int_0^t (C + L)dt\right]0.85K_7 dt + x_{51}(0)\right\}
\end{aligned}\right\} \qquad (2)$$

where $x_{11}(0)$, $x_{12}(0)$, . . . are the initial values of these quantities.

The functions A, B, C, Y_1, Y_4, b are functions of time which can be considered as *control variables* as opposed to the state variables x_{ij}. The second major feature of the model is the set of values or bounds for these control variables which will satisfy the restraints imposed upon the system, such as

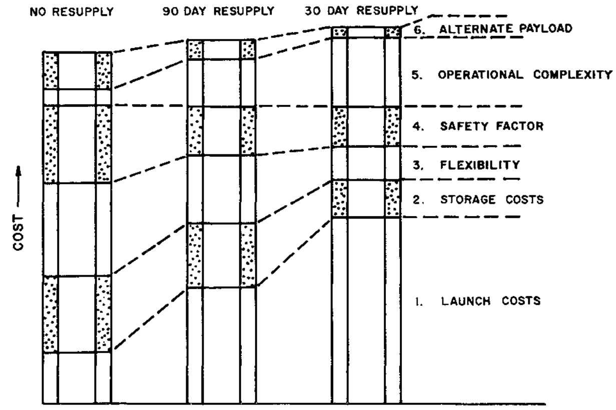

Fig. 1 Cost comparison of mission modes.

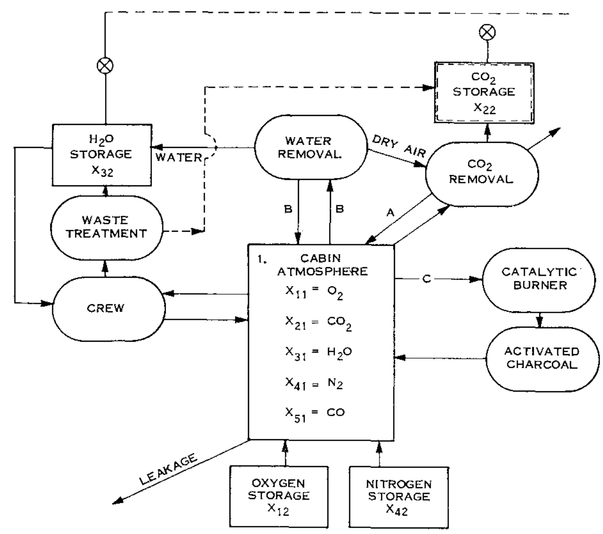

Fig. 2 Basic system.

INGREDIENT / COMPARTMENT / QUANTITY LOCATED IN	1. OXYGEN	2. CARBON DIOXIDE	3. WATER	4. NITROGEN	5. CO	6. CH4
1. ATMOSPHERE	x_{11}	x_{21}	x_{31}	x_{41}	x_{51}	x_{61}
2. STORAGE	x_{12}	x_{22}	x_{32}	x_{42}		

Fig. 3 State variables: basic system.

$$\left.\begin{aligned}
&x_{\min} \leq x_{11} + x_{21} + x_{31} + x_{41} \leq x_{\max} \quad \text{(cabin pressure restraint)} \\
&x_{11\min} \leq x_{11} \leq x_{11\max} \quad \text{(O}_2\text{ partial pressure limits)} \\
&0 \leq x_{21} \leq x_{21\max} \quad \text{(CO}_2\text{ limits)} \\
&x_{31\min} \leq x_{31} \leq x_{31\max} \quad \text{(humidity limits)}
\end{aligned}\right\} \qquad (3)$$

The control variables should be picked so as to optimize certain criterion functions, and there is the implicit restraint that all quantities remain positive.

The control variables mentioned up to this point—the rate at which air is processed through various components, coil temperatures, etc.—must be supplemented by others, some of which are not ordinarily thought of as control variables. Any parameter whose value may be manipulated so as to

Ingredients / Rate of Change Between Atmosphere and:	1. O_2	2. CO_2	3. H_2O	4. N_2	5. CO	6. CH
0. Space	$-Lx_{11}$	$-Lx_{21}$	$-Lx_{31}$	$-Lx_{41}$	$-Lx_{51}$	$-Lx_{61}$
2. Storage	Y_1			Y_4		
3. Waste Unit						
4. Crew	$-K_2$	$.85K_2$	K_3		K_7	
5. Water Removal			$-B(x_{31}-b)$			
6. CO_2 Removal		$-Ax_{21}$				
7. Decontaminant	0		0		$-Cx_{51}$	$-Cx_{61}$

Fig. 4 Rate of change of atmosphere ingredients: basic system.

affect system performance will satisfy our specification of a "control variable." Even the crew's activity level (given by K_2 and K_3) can be considered as a control variable. It is conceivable that, in any emergency situation, all but an essential part of the crew could be put in a low-activity state with drugs, and thereby the life of the system could be extended by a factor of 5 or 6— enough to enable rescue or repair.

The resupply schedule, the central interest of this paper, is according to our definition a control variable. Let $R_i(t)$ be the quantity of the ith ingredient supplied to the system up to time t. The $\dot{R}_i$ of Fig. 4 denotes the rates of resupply. Now $R_i(t)$ terms must be added to expressions (2) for the stored ingredients:

$$\begin{aligned} x_{12} &= -\int_0^t Y_1 dt + x_{12}(0) + R_1(t) \\ x_{32} &= \int_0^t [Bx_{31}(t) - K_4 + K_5 - Bb]dt + x_{32}(0) + R_3(t) \qquad (2') \\ x_{42} &= -\int_0^t K_6 dt + x_{42}(0) + R_4(t) \end{aligned}$$

Within the bounds established by restraints (3), the control variables are chosen so as to optimize certain criteria, such as the system lifetime T, system reliability Q, and the requirements made on the launch vehicle V; V would ordinarily be the weight of the system including that portion of the power supply system weight used to run the life support system. However, questions of launch schedules and package size must also be considered, particularly when determining the best resupply schedule. A precise expression of these quantities requires a definition of the space mission, which is beyond the scope of this paper. However, quite a bit can be done toward optimizing such items as lifetime, weight, and

Ingredient / Rate of Change Between Storage and:	1. O_2	2. CO_2	3. H_2O	4. N_2	5. CO	6. CH_4
0. Space		*	*			
1. Atmosphere	$-Y_1$			$-Y_4$		
3. Waste Unit			K_5			
4. Crew			$-K_4$			
5. Water Removal			$B(x_{31}-b)$			
6. CO_2 Removal		Ax_{21}				
7. Earth (Resupply)	$\dot{R}_1$		$\dot{R}_3$	$\dot{R}_4$		

* = 0 as long as storage capacity is not exceeded

Fig. 5 Rate of change of ingredients in storage.

power consumption, and a choice of controls that accomplish these suboptimizations would almost always be a good choice.

Relative weights α,β,γ must be assigned to T, Q, and V on the basis of their relative importance. Then a criterion function $Z = \alpha T + \beta Q - \gamma V$ is obtained which comprises the third feature of the model. Expressed in terms of the three features, the model is the statement of a problem in control mathematics, very similar in mathematical content to a problem, say, in missile flight control.

Sample Calculations

We assume the following calendar of events:

$t = 0$, valves opened on O_2 and N_2 tanks to fill cabin
t_1, valves shut on O_2 and N_2 containers
t_2, crew enters cabin
t_3, O_2 valve opened to intermediate position
t_3', dehumidifier turned on
t_4, crew becomes less active
t_5, O_2 valve turned lower
t_5', dehumidifier turned lower

Inserting these functions into Eq. (2) leads to a calculation of x_{11}, x_{12}, x_{31}, and x_{32}. The results are shown in Figs. 6 and 7. In Fig. 6 the oxygen in storage is seen to be decreasing. When the curve goes below the minimum reserve supply, the lifetime is ended unless supplies are replenished. The lifetime can be varied by changing the initial value $x_{12}(0)$.

The amount of stored water shown in Fig. 7 increases because the human body produces more water than it consumes. This information will be needed for the calculation of the amount of oxygen and hydrogen which can be produced in a closed system.

Life Support System Using Lithium Carbonate

We now consider an environmental control system that makes use of lithium carbonate to convert CO_2 to O_2. We have in mind a system that is designed essentially as is described in Ref. 1. In addition, H_2O is removed from the atmosphere by two methods: 1) use is made of the silica gel tubes of the CO_2-O_2 exchanger; and 2) air is passed through an air-cooler in which water is condensed and separated from the air (perhaps centrifugally). The dried air is then returned to the atmosphere. The analysis will determine the extent to which system 2 is needed to addend system 1. We assume, as before, that the extra O_2 and N_2 are stored. The prime purpose of the analysis is to determine what amounts of those materials that must be stored initially or obtained by resupply are needed to supplement the regenerated materials on missions of different durations.

A diagram of the CO_2-O_2 converter is given in Fig. 8. Tubes 1a, b, c, d contain silica gel and can be rotated into different positions. Air is dried in 1a at room temperature. Tube 1b contains wet silica gel, and the dry CO_2-free air from the molecular sieve may be passed through this before re-entering the atmosphere. This allows a control in the amount of humidity in the air which returns to the atmosphere from the CO_2-O_2 converter. The silica gel in tube 1c is dried by heating, and tube 1d contains dry silica gel that is being cooled to room temperature. When this tube is cooled, it may be placed in the 1a position. Tubes 2a, b, c are molecular sieves for CO_2 concentration. CO_2 is absorbed from dry air in tube 2a. In tube 2b, the CO_2 is desorbed by heating, the CO_2 passing under desorption pressure into the electrolysis cell. Tube 2c is cooled to room temperature, after which it may be rotated into the 2a position. Of course, more tubes may be placed in the drier or CO_2 concentrator in order to facilitate a faster flow of CO_2 into the electrolysis cell. Part of the purpose of the analysis is to determine what maximum flow rate the system should be able to handle in order to accommodate different system-life and replenishment schedules. This in turn allows estimates of the weight of lithium salt which should be in the lithium converter.

Figure 9 shows the basic aspects of the H_2O cycle. The state variables representing the number of moles of various ingredients in different "compartments" are listed in Fig. 10. The control variables, which may be varied within limits, are listed below. They are nonnegative functions of time:

$A_1(t)$ = rate at which cabin atmosphere enters CO_2-O_2 converter (proportion of total atmospheric volume per hour)

$A_2(t)$ = proportion of H_2O removed from air which passes through CO_2-O_2 converter

$B(t)$ = rate at which cabin atmosphere enters air cooler (proportion of total atmospheric volume per hour)

$Y_1(t)$ = rate at which O_2 enters atmosphere from storage, mole/hr

$Y_4(t)$ = rate at which N_2 enters atmosphere from storage, mole/hr

b = number of moles of H_2O which are not removed from atmosphere when whole cabin atmosphere has passed through separator; function of temperature of air cooler

Other nonnegative parameters of the system are listed below:

K_1 = ratio of O_2 leaving electrolysis cell to CO_2 coming in ($0 < K_2 < 1$, but K_2 should be near 1)

K_2 = rate at which crew consumes O_2, mole/hr

$0.85K_2$ = rate at which crew puts CO_2 into cabin atmosphere

K_3 = rate at which crew puts H_2O into atmosphere, mole/hr

K_4 = proportion of CO_2 going through molecular sieve which is caught ($0 < K_4 < 1$, but K_4 should be near 1)

K_5 = proportion of CO_2 entering electrolysis cell which is not sent back to atmosphere along with O_2($0 < K_5 < 1$, but K_5 should be near 1)

K_6 = rate at which crew produces waste H_2O, mole/hr

K_7 = rate at which crew uses H_2O, mole/hr

L = cabin leakage rate (proportion of total atmospheric volume per hour)

Figure 11 shows the rate at which the atmospheric components are exchanged with the various compartments of the system. Figure 12 shows the rate of change of the various ingredients in storage. The equations for the state variables

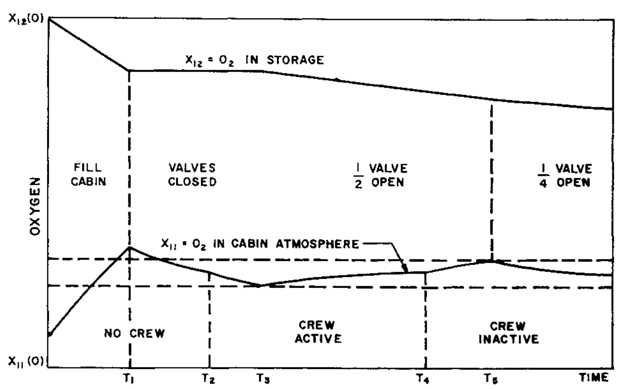

Fig. 6 Variation of oxygen.

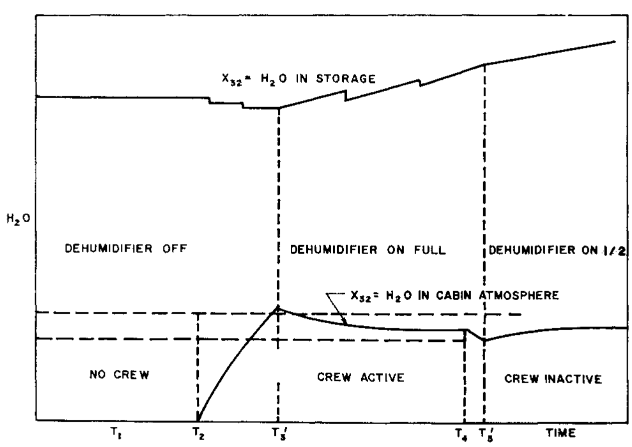

Fig. 7 Variation of water.

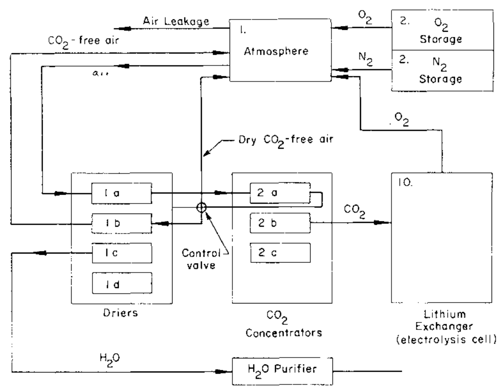

Fig. 8 CO_2 - O_2 converter and O_2, N_2 cycle.

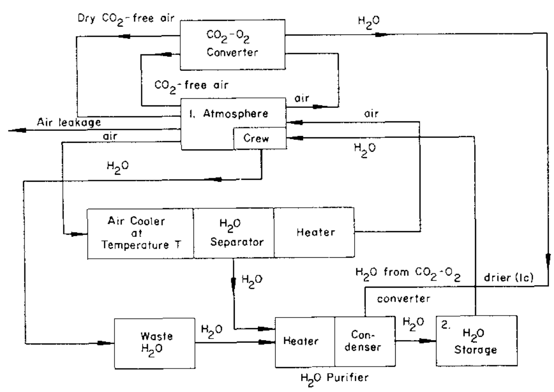

Fig. 9 H_2O cycle.

COMPARTMENT \ INGREDIENT	1. OXYGEN	2. CARBON DIOXIDE	3. WATER	4. NITROGEN	5. CO	6. CH_4	7. CARBON
1. ATMOSPHERE	X_{11}	X_{21}	X_{31}	X_{41}			
2. STORAGE	X_{12}		X_{32}	X_{42}			
3. WASTE			X_{33}				
4. CO_2 - O_2 CONVERTER							X_{74}

Fig. 10 Notation used to represent the number of moles of various ingredients in different locations.

Rate of change between atmosphere and / Ingredient	1. Oxygen	2. Carbon dioxide	3. Water	4. Nitrogen
1. Space	$-Lx_{11}$	$-Lx_{21}$	$-Lx_{31}$	$-Lx_{41}$
2. Storage	$Y_1(t)$			$Y_4(t)$
3. Crew	$-K_2$	$.85K_2$	K_3	
4. CO_2-O_2 converter	$K_1K_4A_1(t)x_{21}$	$-K_4K_5A_1(t)x_{21}$	$-A_1(t)A_2(t)x_{31}$	
5. H_2O removal unit			$-B(t)x_{31}+B(t)b$	

Fig. 11 Rate of change of atmospheric ingredients.

are

$$\left.\begin{aligned}
\dot{x}_{11} &= K_1K_4A_1(t)x_{21} - Lx_{11} - K_2 + Y_1(t)\\
\dot{x}_{12} &= -Y_1(t) + \dot{R}_1(t)\\
\dot{x}_{21} &= -[K_4K_5A_1(t) + L]x_{21} + 0.85K_2\\
\dot{x}_{31} &= -[B(t) + L + A_1(t)A_2(t)]x_{31} + B(t)b + K_3\\
\dot{x}_{32} &= [B(t) + A_1(t)A_2(t)]x_{31} - B(t)b + K_6 - K_7 + \dot{R}_3(t)\\
\dot{x}_{41} &= -Lx_{41} + Y_4(t)\\
\dot{x}_{42} &= -Y_4(t) + \dot{R}_4(t)\\
\dot{x}_{74} &= K_4K_5A_1(t)x_{21}
\end{aligned}\right\} \tag{4}$$

The amount of carbon accumulating in the electrolysis cell x_{74} is equal to the number of moles of CO_2 taken out of the atmosphere by the CO_2-O_2 converter. A record of x_{74} is kept because it might be desirable to change the electrodes when the carbon deposit exceeds a certain amount.

The solution to (4) may be written as

$$\left.\begin{aligned}
x_{11}(t) &= \frac{-K_2}{L} + (\exp[-Lt])\left(x_{11}(0) + \frac{K_2}{L} + \int_0^t [K_1K_4A_1(t)x_{21}(t) + Y_1(t)]\exp[Lt]dt\right)\\
x_{12}(t) &= x_{12}(0) - \int_0^t Y_1(t)dt + R_1(t)\\
x_{21}(t) &= \left(\exp\left[-K_4K_5\int_0^t A_1(t)dt - Lt\right]\right)\times \left(x_{21}(0) + 0.85K_2\int_0^t \exp\left[K_4K_5\int_0^t A_1(t)dt + Lt\right]dt\right)\\
x_{31}(t) &= \left(\exp\left[-\int_0^t [B(t) + A_1(t)A_2(t)]dt - Lt\right]\right)\times \left(x_{31}(0) + \int_0^t [B(t)b + K_3]\times \exp\left[\int_0^t [B(t) + A_1(t)A_2(t)]dt + Lt\right]dt\right)\\
x_{32}(t) &= x_{32}(0) + (K_6 - K_7)t + \int_0^t \{[B(t) + A_1(t)A_2(t)x_{31}(t)] - B(t)]b\}dt + R_3(t)\\
x_{41}(t) &= (\exp[-Lt])\left(x_{41}(0) + \int_0^t Y_4(t)\exp[Lt]dt\right)\\
x_{42}(t) &= x_{42}(0) - \int_0^t Y_4(t)dt + R_4(t)\\
x_{74}(t) &= x_{74}(0) + K_4K_5\int_0^t A_1(t)x_{21}(t)dt
\end{aligned}\right\} \tag{5}$$

Stability and Weight-Power Considerations: Lithium Carbonate System

Stability and the Case of Satisfactory Steady State

Let us suppose that the system has been chosen so that the maximum and minimum values for each control variable are given, for example,

$$A_{1\min} \leq A_1(t) \leq A_{1\max} \qquad \text{all } t > 0 \tag{6}$$

This range is determined by mechanical limitations of the system and the maximum speed at which we are willing to deplete our supply of stored quantities. This latter, in turn, may be determined by a cost of replenishing stored supplies from earth.

We wish to consider what happens to the state of the system as $t \to +\infty$. Assuming that the control variables do not approach zero as $t \to \infty$ and applying L'Hospital's rule, we calculate

$$\left.\begin{aligned}
\lim_{t\to\infty} x_{21}(t) &= \lim_{t\to\infty} \frac{0.85K_2}{K_4K_5A_1(t) + L}\\
\lim_{t\to\infty} x_{11}(t) &= \lim_{t\to\infty} \frac{K_1K_4A_1(t)x_{21}(t) + Y_1(t) - K_2}{L}\\
&= \lim_{t\to\infty}\left(\frac{0.85K_1K_2K_4A_1(t)}{L(K_4K_5A_1(t) + L)} + \frac{Y_1(t) - K_2}{L}\right)\\
\lim_{t\to\infty} x_{31}(t) &= \lim_{t\to\infty} \frac{B(t)b + K_3}{B(t) + A_1(t)A_2(t) + L}\\
\lim_{t\to\infty} x_{41}(t) &= \lim_{t\to\infty} \frac{Y_4(t)}{L}
\end{aligned}\right\} \tag{7}$$

If the control variables are held constant sufficiently long, we may read directly from (7) the atmospheric composition that is approached as time increases (i.e., the steady state). This equilibrium composition, but not the speed of approach, is independent of the initial conditions and may be varied to deal with emergencies.

The atmospheric components of our system have stable solutions in the sense that as time increases, finite limits (which depend on the values of the control variables) are approached. In choosing an optimum system, however, we need more than stability, for it is possible that the limiting values of the x_{i1} are such that the atmosphere will not support life. In this case, we would be interested in knowing how long such a system will support life, assuming that it is initially in a state that supports life. In all cases, we are interested in knowing the ability of the system to restore the atmosphere to a proper balance if, in the event of an emergency or unexpected activity of the crew, the atmospheric components are suddenly thrown out of a tolerable range.

Suppose we begin with initial conditions $x_{i1}(0)$, $i = 1, 2, 3, 4$, which lie in the tolerable range (3). Let

$$\left.\begin{aligned}
L_{11}(t) &= \frac{K_1K_4A_1(t)x_{21} + Y_1(t) - K_2}{L}\\
L_{21}(t) &= \frac{0.85K_2}{K_4K_5A_1(t) + L}\\
L_{31}(t) &= \frac{B(t)b + K_3}{B(t) + A_1(t)A_2(t) + L}\\
L_{41}(t) &= \frac{Y_4(t)}{L}
\end{aligned}\right\} \tag{8}$$

Substitution of (8) into (4) shows that $x_{11}(t)$, $x_{21}(t)$, $x_{31}(t)$, and $x_{41}(t)$ move directly toward their respective $L_{i1}(t)$ values at all times t. Let $L_{i1}{}^M$ be the maximum value of L_{i1} when the control variables are varied between their allowable ranges (and $x_{21\min} \leq x_{21} \leq x_{21\max}$ in the expression for L_{11}), and let $L_{i1}{}^m$ be the corresponding minimum value.

If the initial states are tolerable and the $L_{i1}(t)$ are tolerable for all allowable controls, i.e.,

$$x_{i1\min} \leq L_{i1}{}^m \qquad L_{i1}{}^M \leq x_{i1\max} \tag{9}$$

then all $x_{i1}(t)$ will be tolerable [satisfy the individual restraints of (3)].

Similarly, if

$$x_{\min} \leq L_{11}{}^m + L_{21}{}^m + L_{31}{}^m + L_{41}{}^m \tag{9a}$$

and if

$$x_{\max} \geq L_{11}{}^M + L_{21}{}^M + L_{31}{}^M + L_{41}{}^M \tag{9b}$$

then the atmospheric pressure will remain within the prescribed bounds for all t until the stored quantities are exhausted.

Choice of the extreme values of the control variables as in (6) so that the (9, 9a, and 9b) hold assures us that the atmosphere remains within tolerable bounds either until the stored quantities are exhausted or, in the event of resupply, indefinitely. The choice of the total life support system depends on the choice of the extreme and average values of the control variables. The choice of these values determine the extent to which stored quantities vs regenerated quantities are used. It is possible to express the weight of the whole system in terms of these extreme and average values:

$$W = W(A_1{}^m, A_2{}^m, B^m, \ldots; A_{1\text{av}}, A_{2\text{av}}, \ldots; A_1{}^M, A_2{}^M, B^M, \ldots) \tag{10}$$

The next problem is to choose the extreme and average control values satisfying (9, 9a, and 9b) such that W is minimized. The choice of the extreme values of the controls, however, should take into account the ability of the system to restore the atmospheric components to an allowable range in the event that they have suddenly moved out of this range.

Restoring Ability

Suppose that, because of accident, the atmospheric composition is suddenly out of the tolerable range (3). We are interested in the ranges of the control variables which will have to be built into the environmental control system in order for the system to be capable of restoring the atmosphere to normal. For example, assume that at some time t_1, the CO_2 content of the atmosphere is too high: $x_{21}(t_1) > x_{21\max}$. If the steady-state points, $L_{21}(t)$, lie above $x_{21\max}$ for all values of the control $A_1(t)$, it may be seen from Eqs. (4) and (5) that $x_{21}(t)$ will remain above $x_{21\max}$ for $t > t_1$ no matter how the control A_1 is varied. The situation is illustrated in Fig. 13. Thus, if the steady state is not tolerable, the system has no restoring ability with respect to CO_2. A similar situation, with some modification, holds in the case of the H_2O content of the atmosphere. Such a situation is a main disadvantage of a system whose steady-state points are not tolerable for life support.

INGREDIENT / RATE OF CHANGE BETWEEN AND STORAGE	1. OXYGEN	2. CARBON DIOXIDE	3. WATER	4. NITROGEN
1. SPACE			•	
2. ATMOSPHERE	$-Y_1(t)$			$-Y_4(t)$
3. CREW			$Y_6 - K_7$	
4. CO_2-O_2 CONVERTER			$A_1(t)A_2(t)x_{31}$	
5. H_2O REMOVAL UNIT			$B(t)x_{31} - B(t)b$	
10. EARTH (RESUPPLY)	$\dot{R}_1(t)$		$\dot{R}_3(t)$	$\dot{R}_4(t)$

• = ZERO UNLESS STORAGE CAPACITY IS EXCEEDED

Fig. 12 Rate of change of ingredients in storage.

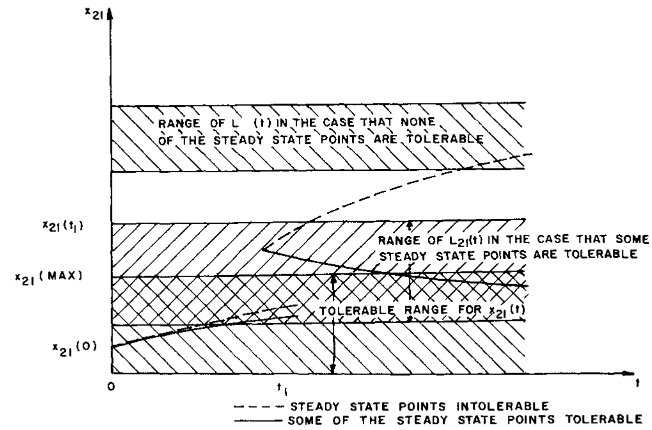

Fig. 13 Behavior of x_{21} (t) if at time t_1 the CO_2 content suddenly becomes too high.

Suppose that, at time t_1,

$$x_{21}(t_1)/x_{21\max} = K > 1$$

and that, at least for large values of A_1, the steady-state points for CO_2 are tolerable. Let A_1 be fixed at such a value, and let us calculate the time $T_{21}(K)$ that it takes for the CO_2 content of the atmosphere to again become tolerable. $T_{21}(K)$ is the smallest value of $t > t_1$ for which the inequality

$$\frac{(0.85)K_2}{K_4K_5A_1 + L} + (\exp[-K_4K_5A_1t - Lt]) \times \left(x_{21}(t_1) - \frac{(0.85)K_2}{K_4K_5A_1 + L}\right) \leq x_{21\max}$$

holds, as may be seen from (5). From this is calculated

$$T_{21}(K) = \frac{1}{K_4K_5A_1 + L} \log\left[\frac{K(K_4K_5A_1 + L)x_{21\max} - 0.85K_2}{(K_4K_5A_1 + L)x_{21\max} - 0.85K_2}\right]$$

provided that $(K_4K_5A_1 + L)x_{21\max} - 0.85K_2 > 0$ [i.e., $\dot{x}_{21} < 0$, as is seen from (4)]. The equation shows that, the larger the value $A_1{}^M$ which the environmental control system is designed to handle, the larger will be the restoring capability of the system with respect to CO_2 in the event of an emergency.

A similar calculation for the H_2O gives

$$T_{31}(K) = \frac{1}{B + A_1A_2 + L} \log\left[\frac{K(B + A_1A_2 + L)x_{31\max} - Bb - K_3}{(B + A_1A_2 + L)x_{31\max} - Bb - K_3}\right]$$

where $x_{31}(t_1)/x_{31\max} = K > 1$.

If in an emergency the O_2 content becomes too low, the best remedy is to let more oxygen into the cabin from storage. Since the converter always produces O_2 at a slower rate than the crew's consumption, increasing the rate at which the CO_2-O_2 converter operates to meet this emergency gains nothing in the way of time or in stored O_2 conserved. The amount of O_2 stored must take into account the number and nature of emergencies we are prepared to meet.

If the atmospheric O_2 content becomes too high, our system has no remedy other than to channel the O_2 from the CO_2-O_2 converter into storage, cease allowing stored oxygen into the cabin, and wait. The situation is similar if the H_2O content of the atmosphere becomes too low. In these events, it is easy to calculate the time during which the components are out of the tolerable range. There should be no loss of nitrogen except due to cabin leakage. The amount of stored nitrogen carried should take into account possible anticipated emergencies involving leakage.

In general, requiring that the system has certain emergency restoring capabilities places design specifications on the extreme values of the controls (e.g., $A_1{}^M$, $Y_1{}^M$, etc.). These extreme values are readily calculated when the restoring capabilities are specified.

Minimizing Weight for Steady-State Points

To get a concrete idea of how we might minimize the weight W [Eq. (10)] of the environmental control system, let us take a particular example to see what the inequalities (9) imply about the ranges of the control variables. Rather than using (9a) and (9b), we shall suppose that the $x_{i1\max}$ and $x_{i1\min}$ have been chosen to satisfy the top equation in (3). The leakage rate L will, in general, be quite small. For a cabin whose size is about 1000 ft^3, a leakage rate of about 12.9 lb/month is not an unreasonable design goal. If we assume that the cabin is kept at approximately standard temperature and pressure, this gives

$$L = 0.00371 \text{ (proportion of atm/day)} = 0.000155 \text{ (proportion of atm/hr)}$$

Let us get some specific values for $x_{i1\max}$ and $x_{i1\min}$ for the case of a two-man crew in a 1000-ft^3 cabin. Suppose that we wish to keep the oxygen partial pressure between 100 and 400 mm Hg and the CO_2 partial pressure less than 6 mm Hg. From the gas law $PV = nRT$, we get $n/p = 1.56$ mole/mm Hg, assuming standard temperature and pressure, giving

$$156 \leq x_{11} \leq 624 \text{ mole} \qquad 9 \leq x_{21} \leq 9.36 \text{ mole} \qquad (11)$$

If the humidity is to remain between 40 and 60%, then

$$10.9 \leq x_{31} \leq 16.4 \text{ mole} \qquad (12)$$

To avoid computations with the nitrogen, we shall assume that it is released from storage at a rate necessary to keep the total atmospheric pressure within tolerable bounds while (11) and (12) are maintained. One can readily calculate values of x_{41} as need arises.

Now, in order to achieve tolerable steady-state points for the atmospheric CO_2 and H_2O, we obtain from (9, 11, and 12)

$$0 \leq L_{21} \leq 9.36 \qquad 10.9 \leq L_{31} \leq 16.4$$

or

$$\left.\begin{aligned} 0 &\leq \frac{0.85K_2}{K_4K_5A_1(t) + L} \leq 9.36 \\ 10.9 &\leq \frac{B(t)b + K_3}{B(t) + A_1(t)A_2(t) + L} \leq 16.4 \end{aligned}\right\} \qquad (13)$$

We take $K_2 = 60/24$ moles/hr, $K_3 = 3.78$ moles/hr, $K_4 = 1$, $K_5 = 0.9$, and $L \approx 0$. Then (13) gives

$$A_1(t) > 0.252 \text{ (proportion of the atm/hr)} \qquad (14a)$$

$$\left.\begin{aligned} [1 - (b/10.9)]B(t) + A_1(t)A_2(t) &\leq 0.0378 \\ 0.0231 \leq [1 - (b/16.4)]B(t) + A_1(t)A_2(t) & \end{aligned}\right\} \qquad (14b)$$

(14a) gives the minimum value of $A_1(t)$ which must be maintained to assure that the steady-state points for CO_2 remain tolerable. If, for example, the CO_2 removal unit is not to be run at all times, perhaps for purposes of power scheduling, then, the average value of $A_1(t)$, when it is operating, would be larger than 0.0252. The control $A_2(t)$ may be varied quite freely between 0 and a number close to 1, say 0.8 or 0.9. The variables $B(t)$ and b in (14b) are associated with the H_2O removal unit, which supplements the H_2O removing done in the CO_2 removal unit. (14) is easily satisfied by taking $B(t) = b = 0$ and suitable values for $A_2(t)$. This indicates that the additional H_2O removal unit is not necessary and that the removal of atmospheric H_2O may well be integrated into the CO_2 removal unit.

For purposes of analyzing the oxygen steady state, let us introduce an additional control variable to the first equation of (4). We wish to see, from the point of view of weight, what is the extent to which the CO_2-O_2 converter should be used as a source of oxygen in preference to initially stored oxygen. Introducing a new control A_3, we rewrite the first equation of (4):

$$\dot{x}_{11} = K_1K_4A_3A_1(t)x_{21} - Lx_{11} - K_2 + Y_1(t) \qquad (15)$$

When A_3 is zero, Eq. (15) gives the atmospheric oxygen change under the assumption that there is no CO_2-O_2 converter. After the CO_2 is removed from the atmosphere at a rate proportional to A_1, it is simply disposed of in space. When A_3 is one, there is full use of the CO_2-O_2 converter. A_3 is not a control which the crew can vary, but rather gives information concerning the best design of the environmental control system. For example, if for a mission of some fixed duration the weight of the system is minimized when A_3 is quite small, say $\frac{1}{10}$, and $Y_{1\text{av}}$ quite large, then this indicates that to a large specified degree it is wiser to use stored oxygen in preference to the CO_2-O_2 converter.

With the introduction of the control A_3, the expression for $L_{11}(t)$ now becomes

$$L_{11}(t) = \frac{K_1K_4A_1(t)A_3x_{21}(t) + Y_1(t) - K_2}{L}$$

In order to get a rough notion of how A_1, A_3, and Y_1 must be related in order to obtain tolerable steady-state points for oxygen, let us assume some average values $x_{21\text{av}}$ for the atmospheric CO_2 concentration, say 5.46 mole (this corresponds to 3.5 mm Hg at standard temperature and pressure). Thus, for tolerable oxygen steady-state points, we get from (9) and (11)

$$156 \leq \frac{K_1K_4A_1(t)A_3x_{21\text{av}} + Y_1(t) - K_2}{L} \leq 624$$

or, in time units of one day,

$$156 \leq \frac{(0.9)(1)(5.46)A_1A_3 + Y_1 - 60}{0.00371} \leq 624 \qquad (16)$$

Here we have taken $K_1 = 0.9$, $K_4 = 1$, $K_2 = 60$, and $L = 0.00371$. From (16), one may calculate

$$60.58 \leq (4.92)A_1A_3 + Y_1 \leq 62.31 \qquad (17)$$

This is a long-range average relationship between the controls

Table 1 Weight interpretation of the terms appearing in Eq. (25)

$C_1A_{1\text{av}}$	Heat removal requirement of fan
$C_2A_3A_{1\text{av}}$	Weight, power requirement, and heat removal requirement of electrolysis cell in CO_2-O_2 converter
$C_3A_1{}^M$	Power and weight requirement of fan; also weight of silica gel and molecular sieves
C_4B_{av}	Heat removal requirement of fan and H_2O separator
C_5B^M	Weight and power requirement for fan and H_2O separator
$C_6Y_{1\text{av}}T_0$	Weight required to store enough O_2 to meet average daily needs of crew
$C_7T_0(Y_1{}^M - Y_{4\text{av}})$	Weight required to store enough extra O_2 to meet possible emergencies; C_7 takes into account the average number of emergencies allowed for in a day
$C_8Y_{4\text{av}}T_0$	Weight required to store enough N_2 to meet average daily needs
$C_9T_0(Y_4{}^M - Y_{4\text{av}})$	Weight required to store enough extra N_2 to meet possible emergencies; C_9 takes into account the average number of emergencies allowed for in a day

A_1, A_3, and Y_1 necessary to obtain tolerable oxygen steady-state points. To deal with oxygen emergencies, we must add a safety margin to the amount of oxygen in storage.

To summarize, in order to have tolerable steady-state points, the lithium carbonate system being considered must be designed in such a fashion that the control variables B, Y_1, A_1, A_2, A_3 satisfy

$$A_{1av} \geq A_1^m \geq 6.05 \tag{18a}$$

$$[1 - (b/10.9)]B_{av} + A_{1av}A_{2av} \leq 0.908$$
$$0.555 \leq [1 - (b/16.4)]B_{av} + A_{1av}A_{2av} \tag{18b}$$

$$60.58 \leq (4.92)A_3A_{1av} + Y_{1av} \leq 62.31 \tag{18c}$$

Here the unit of time is taken to be one day, and we have simply rewritten Eqs. (14) and (17). The problem of weight considerations now becomes that of minimizing a weight function of the control variables under the restriction of (18). Equation (18) may be satisfied by eliminating the addended water separator unit (i.e., take $B = b = 0$) and choosing suitable values for A_2. In addition to (18), there are requirements on the system's ability to restore an intolerable atmospheric composition to a tolerable one. In a fashion indicated earlier, these are realized in the form of certain specified values for A_1^M, B^M, Y_1^M, etc. As a first approximation, we may write the weight of the lithium carbonate system in the form

$$W = C_1A_{1av} + C_2A_3A_{1av} + C_3A_1^M + C_4B_{av} + C_5B^M + C_6Y_{1av}T_0 + C_7T_0(Y_1^M - Y_{1av}) + C_8Y_{4av}T_0 + C_9T_0(Y_4^n - Y_{4av}) \tag{19}$$

where T_0 is the total mission time, and the C_i's are constants that can be calculated from engineering data. The physical interpretation of the terms in (19) is given in Table 1. For a given mission time T_0, the optimum values for the control variables, from the weight point of view, in the lithium carbonate system may be found by minimizing W in (19) under the restriction in (18).

Reference

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Analysis of Stored Gas Pressurization Systems for Propellant Transfer

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The choice of a rocket propellant feed system is, by necessity, based on weight and reliability. Among the methods available for propellant transfer during a space mission with many vehicle maneuvers, the most attractive, primarily due to reliability and simple restart procedure, is expulsion by pressurization with stored, low molecular weight, inert gas. In this paper the performance analysis method for such a propellant transfer-system design is presented. Theoretical equations are developed from fundamental thermodynamic and heat-transfer concepts.

Nomenclature

A_E = heat-transfer area, ft²
c_P = heat capacity at constant pressure, Btu/lbm-°R
c_V = heat capacity at constant volume, Btu/lbm-°R
d = differential operator
D = diameter, ft
g = acceleration, ft/sec²
h = specific enthalpy, Btu/lbm
h_{fg} = latent heat of vaporization of the propellant, Btu/lbm
h_c = heat-transfer film coefficient, Btu/sec-ft²-°R
J = mechanical equivalent of heat, 778.2 ft-lb/Btu
k = thermal conductivity, Btu-ft/sec-ft²-°R
m = mass, lbm
Q = heat, Btu
P = pressure, lb/ft²
R = specific gas constant, ft-lb/lbm-°R
T = temperature, °R
ΔT = difference between pressurant and bottle wall temperatures, °R
U = internal energy, Btu
U_e = overall heat-transfer coefficient, Btu/sec-ft²-°R
V = volume, ft³
$\bar{V}$ = mean velocity, fps
Z = compressibility factor
β = coefficient of thermal expansion, 1/°R
π = partial pressure, lb/ft²
ρ = density, lbm/ft³
μ = viscosity, lbm/ft-sec
θ = time, sec

Subscripts

B = storage bottle
C = coil
E = heat exchanger
G = pressurant
I = inside

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